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Corrigendum to "Extended Formulations and Branch-and-Cut Algorithms for the Black-and-White Traveling Salesman Problem" [European Journal of Operational Research, 262(3) 2017, 908–928]

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A B S T R A C T

In a personal communication with Ruslan Sadykov from Inria, France, we found an implementation error in our code for importing the benchmark instance set MUT leading to wrong numerical results in our original article (Gouveia, Leitner, and Ruthmair, 2017). In this corrigendum we provide corrected results for all experiments on instance set MUT. The general findings and conclusions drawn from the results however do not change.

Tables 3, 4, 7, and 8 in the original article (Gouveia, Leitner, & Ruthmair, 2017) have to be replaced with the corresponding tables in this corrigendum. Additionally, several numbers of solved instances in a paragraph in Section 7.3 have to be modified as stated below (we only cite relevant parts of the original article):

7.3. Branch-and-cut result overview

...

Finally, note that a direct comparison to the results of the branch-and-price approach in Muter (2015) is only possible for instance set MUT and when $\gamma = 1.0$: The best method from Muter (2015) is able to solve 74 instances within 10800 seconds on a hardware similar to ours while our approach based on model PD$^+$ solves 163 instances within 7200 seconds. Since we have different values of $L_{\max}$ (the ones in Muter (2015) have been obtained by a heuristic) the instances for restricted L values are not the same. Generating the $L_{\max}$ values in the same way as we did for the other instances, it seems that we solve less instances than Muter (2015) ($\gamma = 0.7$: 120 by Muter, 118 by DD$^+$; $\gamma = 0.8$: 115 by Muter, 93 by PS$^+$). To this end, we stress that the (likely) different values of $L_{\max}$ may significantly influence the overall performance, thus avoiding significant conclusions from this comparison. We also note, that the distance-dependent models heavily depend on the range of the distance values. If we for example would divide all the distance values for the MUT instances by 100 and round up the resulting values, we obtain much better results for the DD$^+$ model ($\gamma = 0.7$: 155; $\gamma = 0.8$: 151).

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Table 3
Number of instances (out of 20 in each row) solved by our branch-and-cut algorithms based on different models within 7200 seconds and 5 GB memory. Bold values denote the best algorithms for a set and a model type.

Set	Q/α	β	γ	PS		PD			PDPS		
					+		+	++		+	++
GLR1	4	1.00	0.6	3	**4**	**7**	**7**	**7**	**5**	**5**	**5**
			0.8	**5**	**5**	**16**	**16**	**16**	11	**12**	**12**
			1.0	**5**	**5**	17	**19**	**19**	15	**17**	**17**
		1.33	0.6	5	**6**	**12**	**12**	**12**	**6**	5	5
			0.8	11	**13**	19	**20**	**20**	14	**16**	15
			1.0	14	**15**	**20**	**20**	**20**	17	**19**	**19**
		1.67	0.6	9	**10**	**19**	18	**19**	**8**	**8**	**8**
			0.8	**17**	**17**	**20**	**20**	**20**	**19**	**19**	**19**
			1.0	**18**	**18**	**20**	**20**	**20**	**20**	**20**	**20**
	8	1.00	0.6	**6**	5	**6**	**6**	**6**	**6**	**6**	**6**
			0.8	**8**	**8**	10	**11**	10	**9**	**9**	**9**
			1.0	9	**10**	14	**16**	**16**	11	**14**	**14**
		1.33	0.6	**10**	9	**10**	**10**	9	6	**8**	7
			0.8	**16**	15	18	**20**	**20**	11	**13**	12
			1.0	**19**	**19**	19	**20**	**20**	17	**19**	**19**
		1.67	0.6	9	**10**	**11**	**11**	**11**	6	**7**	6
			0.8	**19**	18	**19**	**19**	**19**	**13**	**13**	**13**
			1.0	**19**	**19**	**20**	**20**	**20**	18	**19**	18
GLR2	4	1.00	0.6	**1**	0	5	**6**	**6**	**5**	**5**	**5**
			0.8	1	**3**	**14**	**14**	**14**	9	**12**	**12**
			1.0	**1**	**1**	15	**18**	17	11	**14**	**14**
		1.33	0.6	**4**	**4**	**6**	**6**	**6**	**5**	**5**	**5**
			0.8	**5**	**5**	12	**19**	17	8	**9**	**9**
			1.0	**5**	**5**	17	**19**	18	10	**14**	**14**
		1.67	0.6	**4**	**4**	8	**9**	**9**	**6**	**6**	**6**
			0.8	**6**	5	11	**16**	15	8	**9**	**9**
			1.0	**6**	**6**	17	**19**	**19**	10	**13**	**13**
	8	1.00	0.6	**3**	**3**	**4**	**4**	**4**	**5**	**5**	**5**
			0.8	**5**	**5**	10	**11**	**11**	6	**8**	7
			1.0	**5**	**5**	12	**16**	**16**	9	**12**	11
		1.33	0.6	3	**5**	**4**	**4**	**4**	**5**	4	4
			0.8	**5**	**5**	**8**	**8**	**8**	5	**6**	**6**
			1.0	**5**	**5**	10	**13**	**13**	9	**10**	**10**
		1.67	0.6	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
			0.8	**6**	**6**	9	**12**	**12**	6	**8**	**8**
			1.0	**7**	**7**	12	**14**	**14**	9	**11**	**11**
MUT	0.2	1.00	0.7	19	**20**	**20**	**20**	**20**	**20**	**20**	**20**
			0.8	13	**14**	**14**	**14**	**14**	13	**14**	**14**
			1.0	4	**5**	15	**16**	**16**	12	**14**	**14**
		1.33	0.7	**8**	**8**	**9**	**9**	**9**	7	**8**	**8**
			0.8	**6**	**6**	**8**	**8**	**8**	**6**	**6**	**6**
			1.0	11	**12**	17	**18**	17	11	**15**	**15**
		1.67	0.7	**6**	**6**	**8**	**8**	**8**	6	**7**	**7**
			0.8	8	**9**	10	**11**	**11**	**6**	**6**	**6**
			1.0	**17**	**17**	18	**20**	**20**	**16**	15	15
	0.3	1.00	0.7	18	**19**	**17**	**17**	**17**	**17**	**17**	**17**
			0.8	**7**	**7**	6	**7**	**7**	**6**	**6**	5
			1.0	14	**16**	16	**17**	16	**11**	**11**	**11**
		1.33	0.7	7	**8**	**7**	**7**	**7**	**7**	**7**	**7**
			0.8	**9**	**9**	**9**	**9**	**9**	**7**	**7**	**7**
			1.0	**19**	**19**	**20**	**20**	18	**14**	**14**	**14**
		1.67	0.7	**8**	**8**	**7**	**7**	**7**	**7**	6	6
			0.8	**10**	9	**8**	**8**	**8**	**6**	**6**	**6**
			1.0	**20**	**20**	**20**	**20**	**20**	**19**	17	16
	0.4	1.00	0.7	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**
			0.8	**12**	**12**	**11**	**11**	**11**	**12**	11	11
			1.0	**15**	**15**	**16**	**16**	**16**	13	**14**	**14**
		1.33	0.7	**9**	**9**	**8**	**8**	**8**	**8**	**8**	**8**
			0.8	**11**	**11**	**10**	**10**	**10**	**8**	**8**	**8**
			1.0	**18**	**18**	**17**	**17**	**17**	**14**	**14**	13
		1.67	0.7	8	**10**	**7**	**7**	**7**	**5**	**5**	**5**
			0.8	15	**16**	**12**	11	11	**8**	**8**	**8**
			1.0	**20**	**20**	**19**	**19**	**19**	**19**	18	18
			Total	611	**628**	805	**853**	843	641	**687**	677

Table 4
Number of instances (out of 20 in each row) solved by our branch-and-cut algorithms based on different models within 7200 seconds and 5 GB memory. Bold values denote the best algorithms for a set and a model type.

Set	Q/α	β	γ	DD			DDPS			PDDD			3PD		
					+	++		+	++		+	++		+	++
GLR1	4	1.00	0.6	**5**	**5**	**5**	**5**	**5**	**5**	8	**11**	**11**	**14**	**14**	**14**
			0.8	5	**6**	**6**	5	**6**	**6**	10	**13**	12	14	**15**	14
		1.33	0.6	14	**17**	16	**9**	**9**	**9**	12	**18**	16	16	**19**	**19**
			0.8	15	**16**	15	7	**9**	8	16	**19**	18	17	**19**	**19**
		1.67	0.6	18	**20**	**20**	**12**	**12**	**12**	17	**20**	**20**	17	**20**	**20**
			0.8	19	**20**	**20**	**13**	12	12	**20**	**20**	**20**	**20**	**20**	**20**
	8	1.00	0.6	8	**10**	9	5	**7**	**7**	6	**10**	**10**	5	**6**	5
			0.8	7	**8**	**8**	5	6	**7**	7	**9**	8	6	**8**	**8**
		1.33	0.6	13	**16**	15	8	**10**	**10**	11	**13**	**13**	8	11	**12**
			0.8	15	**17**	**17**	**9**	**9**	**9**	14	14	**15**	9	**12**	**12**
		1.67	0.6	16	**20**	19	9	**12**	**12**	13	**14**	**14**	10	**12**	**12**
			0.8	18	**20**	19	**12**	**12**	**12**	17	**18**	17	13	**14**	13
GLR2	4	1.00	0.6	**4**	**4**	**4**	**4**	**4**	**4**	5	**9**	**9**	11	**15**	14
			0.8	**3**	**3**	**3**	**3**	**3**	**3**	8	**13**	12	9	**13**	12
		1.33	0.6	7	**9**	**9**	5	**6**	**6**	10	**11**	**11**	11	**12**	**12**
			0.8	**6**	**6**	**6**	**5**	**5**	**5**	9	9	**10**	10	**12**	11
		1.67	0.6	9	**10**	**10**	5	**7**	**7**	10	13	**14**	12	**15**	**15**
			0.8	**8**	**8**	**8**	5	**6**	**6**	9	**13**	**13**	10	13	**14**
	8	1.00	0.6	6	**9**	8	5	**7**	**7**	6	**9**	8	**5**	**5**	**5**
			0.8	5	**6**	**6**	4	**5**	**5**	6	**7**	6	**5**	**5**	**5**
		1.33	0.6	9	**11**	**11**	5	7	**8**	6	**10**	**10**	5	9	**10**
			0.8	**8**	**8**	**8**	**5**	**5**	**5**	6	**9**	**9**	5	**7**	6
		1.67	0.6	10	**12**	10	6	**8**	**8**	9	**10**	**10**	6	**10**	**10**
			0.8	**9**	**9**	**9**	5	**6**	**6**	8	**9**	**9**	6	**7**	**7**
MUT	0.20	1.00	0.7	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**
			0.8	**14**	**14**	**14**	**14**	**14**	**14**	**14**	**14**	**14**	**14**	**14**	**14**
		1.33	0.7	**12**	**12**	**12**	**12**	**12**	11	**11**	**11**	**11**	10	**11**	**11**
			0.8	**9**	**9**	**9**	**9**	**9**	**9**	**9**	8	8	**9**	7	8
		1.67	0.7	**13**	**13**	**13**	**12**	**12**	11	12	**13**	**13**	**10**	**10**	**10**
			0.8	**10**	**10**	**10**	**9**	7	7	**9**	8	8	**9**	7	7
	0.30	1.00	0.7	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**
			0.8	**7**	**7**	**7**	**7**	**7**	**7**	6	**7**	**7**	**5**	**5**	**5**
		1.33	0.7	9	**10**	**10**	**10**	9	9	**10**	9	9	**10**	9	9
			0.8	**7**	**7**	**7**	**7**	6	6	**6**	**6**	**6**	**7**	6	6
		1.67	0.7	11	**12**	**12**	9	**10**	**10**	8	**9**	**9**	**9**	**9**	**9**
			0.8	**6**	**6**	**6**	5	**6**	**6**	**5**	**5**	**5**	**5**	**5**	**5**
	0.40	1.00	0.7	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**	**19**
			0.8	**12**	**12**	**12**	**12**	**12**	**12**	**12**	**12**	11	**10**	**10**	**10**
		1.33	0.7	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**8**	**8**	**8**
			0.8	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**
		1.67	0.7	6	**7**	**7**	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
			0.8	**9**	8	8	**7**	**7**	**7**	**9**	7	7	**7**	**7**	**7**
			Total	433	**468**	459	346	**366**	365	431	**487**	480	425	**469**	466

Table 7
Comparison of final optimality gaps, CPU times, and numbers of solved and infeasible instances of our branch-and-cut algorithms based on different models for instances with $|V| \in \{20, 40\}$ from set MUT. Bold values denote the best algorithms in a row. (“tl”...time limit reached, “-”...results not available)

$\|V\|$	α	β	γ	Avg. optimality gaps in %					Avg. CPU times in seconds					# Instances solved (inf.) (out of 5)				
				PS+	PD+	DD+	PDDD+	3PD+	PS+	PD+	DD+	PDDD+	3PD+	PS+	PD+	DD+	PDDD+	3PD+
20	0.2	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	-	-	-	-	-	262	591	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			1.0	4.9	**0.0**	-	-	-	3776	**1**	-	-	-	3(0)	**5(0)**	-	-	-
		1.33	0.7	**8.7**	-	-	-	-	1441	77	**0**	1	1	4(4)	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	258	47	**1**	**1**	**1**	**5(3)**	**5(3)**	**5(3)**	**5(3)**	**5(3)**
			1.0	**0.0**	**0.0**	-	-	-	**1**	**1**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	2.8	**0.0**	**0.0**	**0.0**	**0.0**	1490	7	**3**	10	5	4(3)	**5(3)**	**5(3)**	**5(3)**	**5(3)**
			0.8	2.7	**0.0**	**0.0**	**0.0**	**0.0**	1469	**2**	16	28	20	4(2)	**5(2)**	**5(2)**	**5(2)**	**5(2)**
			1.0	**0.0**	**0.0**	-	-	-	109	**1**	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.3	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	-	-	-	-	-	**0**	**0**	1	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			1.0	**0.0**	**0.0**	-	-	-	116	**1**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.33	0.7	-	**13.7**	-	-	-	386	1440	**13**	36	33	**5(5)**	4(4)	**5(5)**	**5(5)**	**5(5)**
			0.8	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**12**	31	108	90	144	**5(2)**	**5(2)**	**5(2)**	**5(2)**	**5(2)**
			1.0	**0.0**	**0.0**	-	-	-	**1**	**1**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	47	45	1	**0**	1	**5(4)**	**5(4)**	**5(4)**	**5(4)**	**5(4)**
			0.8	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	2	**1**	18	24	55	**5(2)**	**5(2)**	**5(2)**	**5(2)**	**5(2)**
			1.0	**0.0**	**0.0**	-	-	-	**0**	**0**	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.4	1.00	0.7	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	3	**1**	59	101	686	**5(4)**	**5(4)**	**5(4)**	**5(4)**	**5(4)**
			0.8	**0.0**	3.4	**0.0**	**0.0**	4.9	**174**	1452	724	1194	1468	**5(4)**	4(3)	**5(4)**	**5(4)**	4(4)
			1.0	**0.0**	**0.0**	-	-	-	2	**1**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.33	0.7	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**3**	5	80	143	377	**5(3)**	**5(3)**	**5(3)**	**5(3)**	**5(3)**
			0.8	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**6**	56	309	313	1435	**5(0)**	**5(0)**	**5(0)**	**5(0)**	**5(0)**
			1.0	**0.0**	**0.0**	-	-	-	**0**	**0**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	82	56	2	**1**	2	**5(4)**	**5(4)**	**5(4)**	**5(4)**	**5(4)**
			0.8	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	6	**2**	21	18	68	**5(1)**	**5(1)**	**5(1)**	**5(1)**	**5(1)**
			1.0	**0.0**	**0.0**	-	-	-	**0**	**0**	-	-	-	**5(0)**	**5(0)**	-	-	-
40	0.2	1.00	0.7	-	-	-	-	-	393	78	**21**	23	99	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	16.7	11.9	**7.6**	10.9	14.7	**2880**	**2880**	**2880**	**2880**	**2880**	**3(3)**	**3(3)**	**3(3)**	**3(3)**	**3(3)**
			1.0	3.2	**0.0**	-	-	-	4841	**43**	-	-	-	2(0)	**5(0)**	-	-	-
		1.33	0.7	9.1	8.6	6.3	10.3	**5.8**	4320	3983	**1523**	1722	1665	2(2)	3(2)	**4(4)**	**4(4)**	**4(4)**
			0.8	13.8	12.2	**7.3**	9.1	8.6	5909	4597	**4062**	5597	6478	1(0)	2(0)	**3(1)**	2(1)	1(0)
			1.0	0.4	**0.0**	-	-	-	1452	**10**	-	-	-	4(0)	**5(0)**	-	-	-
		1.67	0.7	17.0	14.0	**0.0**	**0.0**	1.3	5760	4858	1250	**1210**	2184	1(1)	2(1)	**5(3)**	**5(3)**	4(3)
			0.8	**0.0**	**0.0**	**0.0**	1.2	1.4	2422	**317**	2139	3791	5098	**5(0)**	**5(0)**	**5(0)**	3(0)	2(0)
			1.0	**0.0**	**0.0**	-	-	-	6	**5**	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.3	1.00	0.7	**0.0**	6.4	6.5	4.8	20.1	**525**	1772	1494	1616	2413	**5(4)**	4(4)	4(4)	4(4)	4(4)
			0.8	**4.3**	**4.3**	8.7	8.2	22.0	**4347**	5187	4877	5385	tl	**2(0)**	**2(0)**	**2(2)**	**2(2)**	0(0)
			1.0	**0.0**	**0.0**	-	-	-	400	**15**	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.33	0.7	12.1	12.5	**2.2**	4.0	4.2	4322	4329	**2305**	2912	2962	2(1)	2(1)	**4(3)**	3(3)	3(3)
			0.8	4.3	4.8	**3.8**	4.6	10.8	**4321**	4324	5369	5760	5760	**2(1)**	**2(1)**	**2(1)**	1(1)	1(1)
			1.0	**0.0**	**0.0**	-	-	-	**2**	4	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	12.2	14.1	**0.0**	1.0	12.0	5760	5760	**893**	3073	2340	1(1)	1(1)	**5(3)**	3(3)	4(3)
			0.8	7.8	**7.3**	**7.3**	10.0	10.8	tl	tl	**6720**	tl	tl	0(0)	0(0)	**1(0)**	0(0)	0(0)
			1.0	**0.0**	**0.0**	-	-	-	**3**	4	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.4	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	5.2	**4.2**	8.5	10.6	8.2	**4320**	**4320**	4324	4342	5760	**2(2)**	**2(2)**	**2(2)**	**2(2)**	1(1)
			1.0	1.2	**0.0**	-	-	-	2910	**63**	-	-	-	3(0)	**5(0)**	-	-	-
		1.33	0.7	**4.0**	4.8	6.5	6.6	17.6	**2904**	4320	2905	2965	4320	**3(3)**	2(2)	**3(3)**	**3(3)**	2(2)
			0.8	**1.1**	1.7	5.9	4.7	10.1	**1480**	1883	5760	5760	5760	**4(1)**	**4(1)**	1(1)	1(1)	1(1)
			1.0	**0.0**	**0.0**	-	-	-	**3**	6	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	**0.8**	3.3	3.6	7.3	12.3	**3066**	4400	4874	5762	5761	**4(1)**	2(1)	2(1)	1(1)	1(1)
			0.8	1.9	2.2	**1.1**	1.4	3.1	**1464**	1982	2933	4523	4521	**4(0)**	**4(0)**	3(1)	2(1)	2(1)
			1.0	**0.0**	**0.0**	-	-	-	**1**	2	-	-	-	**5(0)**	**5(0)**	-	-	-

Table 8
Comparison of final optimality gaps, CPU times, and numbers of solved and infeasible instances of our branch-and-cut algorithms based on different models for instances with $|V| \in \{60, 80\}$ from set MUT. Bold values denote the best algorithms in a row. ("tl"...time limit reached, "-"...results not available)

$\|V\|$	α	β	γ	Avg. optimality gaps in %					Avg. CPU times in seconds					# Instances solved (inf.) (out of 5)				
				PS$^+$	PD$^+$	DD$^+$	PDDD$^+$	3PD$^+$	PS$^+$	PD$^+$	DD$^+$	PDDD$^+$	3PD$^+$	PS$^+$	PD$^+$	DD$^+$	PDDD$^+$	3PD$^+$
60	0.2	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	7.5	**4.4**	8.7	9.8	-	**4320**	**4320**	**4320**	**4320**	**4320**	**2(2)**	**2(2)**	**2(2)**	**2(2)**	**2(2)**
			1.0	4.1	**0.0**	-	-	-	tl	**814**	-	-	-	0(0)	**5(0)**	-	-	-
		1.33	0.7	9.9	9.6	**5.4**	9.3	18.6	4491	5760	**3245**	4645	5434	2(1)	1(1)	**3(3)**	2(2)	2(2)
			0.8	12.9	**9.8**	17.5	15.8	33.5	tl	6073	**5792**	5858	5939	0(0)	**1(0)**	**1(1)**	**1(1)**	**1(1)**
			1.0	0.8	**0.0**	-	-	-	5309	**270**	-	-	-	2(0)	**5(0)**	-	-	-
		1.67	0.7	23.2	23.6	**20.3**	22.9	26.2	tl	tl	**4425**	5370	tl	0(0)	0(0)	**2(2)**	**2(2)**	0(0)
			0.8	6.6	**6.2**	10.4	8.6	25.2	tl	**6596**	tl	tl	tl	0(0)	**1(0)**	0(0)	0(0)	0(0)
			1.0	0.4	**0.0**	-	-	-	1520	**130**	-	-	-	4(0)	**5(0)**	-	-	-
	0.3	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	8.2	**8.1**	16.6	17.5	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	2.1	**0.1**	-	-	-	4783	**2534**	-	-	-	2(0)	**4(0)**	-	-	-
		1.33	0.7	**9.1**	11.3	15.0	17.4	-	**5760**	**5760**	**5760**	**5760**	**5760**	**1(1)**	**1(1)**	**1(1)**	**1(1)**	**1(1)**
			0.8	5.8	**5.2**	12.5	15.7	-	**5138**	5560	tl	tl	tl	**2(0)**	**2(0)**	0(0)	0(0)	0(0)
			1.0	0.9	**0.0**	-	-	-	1449	**819**	-	-	-	4(0)	**5(0)**	-	-	-
		1.67	0.7	**10.9**	11.5	14.1	**10.9**	33.1	**5762**	5765	5794	5961	tl	**1(0)**	**1(0)**	**1(0)**	**1(0)**	0(0)
			0.8	4.1	**3.2**	15.4	15.3	-	**3285**	4150	tl	tl	tl	**3(0)**	**3(0)**	0(0)	0(0)	0(0)
			1.0	**0.0**	**0.0**	-	-	-	**10**	12	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.4	1.00	0.7	14.3	**10.1**	-	-	-	**1440**	**1440**	**1440**	**1440**	**1440**	**4(4)**	**4(4)**	**4(4)**	**4(4)**	**4(4)**
			0.8	10.1	**5.3**	-	-	-	**4320**	**4320**	**4320**	**4320**	**4320**	**2(2)**	**2(2)**	**2(2)**	**2(2)**	**2(2)**
			1.0	4.2	**0.1**	-	-	-	3348	**2044**	-	-	-	3(0)	**4(0)**	-	-	-
		1.33	0.7	**8.7**	9.8	17.3	17.3	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			0.8	**6.5**	7.7	19.9	22.7	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	0.8	**0.7**	-	-	-	**2899**	2964	-	-	-	**3(0)**	**3(0)**	-	-	-
		1.67	0.7	**10.7**	11.7	14.2	14.7	22.0	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			0.8	**2.6**	4.7	16.1	14.8	-	**3186**	5851	tl	tl	tl	**3(0)**	1(0)	0(0)	0(0)	0(0)
			1.0	**0.0**	**0.0**	-	-	-	**4**	29	-	-	-	**5(0)**	**5(0)**	-	-	-
80	0.2	1.00	0.7	-	-	**10.5**	**10.5**	-	97	**10**	1440	1440	1440	**5**	**5**	4(4)	4(4)	4(4)
			0.8	13.7	**8.3**	21.7	21.7	-	**1440**	**1440**	**1440**	**1440**	**1440**	**4(4)**	**4(4)**	**4(4)**	**4(4)**	**4(4)**
			1.0	6.8	**1.7**	-	-	-	tl	**5794**	-	-	-	0(0)	**1(0)**	-	-	-
		1.33	0.7	16.7	**16.3**	18.1	18.6	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			0.8	**6.7**	**6.7**	27.5	27.6	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	2.2	**0.3**	-	-	-	5787	**4345**	-	-	-	1(0)	**3(0)**	-	-	-
		1.67	0.7	**12.8**	13.2	16.8	16.4	-	**5760**	**5760**	**5760**	**5760**	**5760**	**1(1)**	**1(1)**	**1(1)**	**1(1)**	**1(1)**
			0.8	**5.1**	5.5	17.9	16.4	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	1.0	**0.0**	-	-	-	3736	**1805**	-	-	-	3(0)	**5(0)**	-	-	-
	0.3	1.00	0.7	2.5	**1.5**	-	-	-	**2668**	2880	2880	2880	2880	**4(4)**	3(3)	3(3)	3(3)	3(3)
			0.8	**7.3**	**7.3**	-	-	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	0.9	**0.7**	-	-	-	**2890**	4326	-	-	-	**4(0)**	3(0)	-	-	-
		1.33	0.7	**9.2**	9.8	22.7	21.1	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			0.8	**4.7**	5.9	-	-	-	tl	tl	tl	tl	tl	**0(0)**	**0(0)**	**0(0)**	**0(0)**	**0(0)**
			1.0	**0.0**	**0.0**	-	-	-	**1618**	3138	-	-	-	**5(0)**	**5(0)**	-	-	-
		1.67	0.7	**10.6**	13.9	23.2	19.4	-	7085	tl	**5921**	tl	tl	**1(0)**	0(0)	**1(1)**	0(0)	0(0)
			0.8	**5.0**	6.6	17.5	18.9	-	**6350**	tl	tl	tl	tl	**1(0)**	0(0)	0(0)	0(0)	0(0)
			1.0	**0.0**	**0.0**	-	-	-	**34**	384	-	-	-	**5(0)**	**5(0)**	-	-	-
	0.4	1.00	0.7	-	-	-	-	-	**0**	**0**	**0**	**0**	**0**	**5(5)**	**5(5)**	**5(5)**	**5(5)**	**5(5)**
			0.8	**5.8**	7.2	-	-	-	**2880**	**2880**	**2880**	**2880**	**2880**	**3(3)**	**3(3)**	**3(3)**	**3(3)**	**3(3)**
			1.0	**0.2**	0.7	-	-	-	**1743**	4511	-	-	-	**4(0)**	2(0)	-	-	-
		1.33	0.7	**7.5**	8.7	-	-	-	**5760**	**5760**	**5760**	**5760**	**5760**	**1(1)**	**1(1)**	**1(1)**	**1(1)**	**1(1)**
			0.8	**3.6**	4.0	-	-	-	**5422**	5760	5760	5760	5760	**2(1)**	1(1)	1(1)	1(1)	1(1)
			1.0	**0.0**	0.1	-	-	-	**585**	3195	-	-	-	**5(0)**	4(0)	-	-	-
		1.67	0.7	**4.7**	7.0	-	-	-	**6517**	tl	tl	tl	tl	**1(0)**	0(0)	0(0)	0(0)	0(0)
			0.8	**0.2**	3.1	-	-	-	**3565**	6056	tl	tl	tl	**4(0)**	1(0)	0(0)	0(0)	0(0)
			1.0	**0.0**	0.2	-	-	-	**467**	1455	-	-	-	**5(0)**	4(0)	-	-	-

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